Modeling Filaments Produced By Magnetically Confined Plasmas

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Physical Problem

Cross-field particle transport in magnetic confinement devices is dominated by intermittent ejection of filamentary structures from the centre of the plasma. These filaments move under the action of an effective gravity force resulting from curvature and magnetic field gradients. They transport hot, dense plasma across a region of open magnetic field lines known as the scrape-off layer towards the surrounding material boundaries. In fusion reactors, large fluxes of plasma impinging on material walls can lead to erosion of the surface and thus limit the device lifetime. Furthermore, liberated surface particles can enter the core of the plasma as impurities and degrade the performance of the reactor. The goal of this research is to uncover the fundamental mechanism for generation of plasma filaments.

Mathematical model

The equations presented in this work have been obtained using an electrostatic drift-fluid model. The model assumes singly charged cold ions and isothermal electrons. The geometry is simplified to a local slab geometry with a uniform magnetic field \( \mathbf{B} = B \hat{z} \). The effects of magnetic curvature and inhomogeneity of \( \mathbf{B} \) are represented through additional terms in the evolution equations as effective gravity acting in the radial direction. The \( x \) and \( y \) coordinates represent respectively the effective radial and poloidal directions. The governing equations consist of the evolution equations for plasma density, \( n \) and plasma vorticity, \( \omega \):

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{v}) = 0, \quad \frac{\partial \mathbf{v}}{\partial t} + \nabla \mathbf{v} \cdot \mathbf{v} = -\nabla \Phi - \frac{\nabla \times \mathbf{B}}{\mu_0} \times \mathbf{v},
\]

where \( \mu_0 \) is the magnetic permeability, \( \nabla \Phi \) is the electric field, \( \mathbf{B} \) is the magnetic field.\(^\text{1}\)

Linear Stability Analysis

We construct a static, steady basic state solution \( (U_0 = 0) \) with the density varying as a function of the radial coordinate only \( (N_i = N_i(x)) \). Introducing \( \theta = \log(n/n_0) \) and non-dimensionalising (1), (2) we derive linearised equations governing perturbations to the basic state

\[
\frac{\partial \omega}{\partial \theta} = - \frac{\partial n}{\partial x} \left( \frac{R_\text{B}}{n} \right) + \frac{\partial \mathbf{v}}{\partial x} \cdot \nabla \omega, \quad \frac{\partial n}{\partial \theta} = - \frac{R_\text{B}}{n} \frac{\partial \mathbf{v}}{\partial x} \cdot \nabla n + \nabla \theta \nabla \theta + 2 \frac{\partial \mathbf{v}}{\partial x} \frac{\partial \mathbf{v}}{\partial x} \nabla \omega
\]

We note that the system of equations (3), (4) resembles the equations governing the classical fluid dynamics problem of 2-dimensional Rayleigh–Bénard convection, see (1), (2), with additional terms in the temperature equation (highlighted in (4)). We note also that in contrast to the classical RBC the background basic state temperature \( \theta(x) \) is nonlinear and satisfies \( \frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial y} = \frac{1}{2} > 0 \). We can exploit the analogy between the two systems to investigate the effects of these extra terms and also provide an interpretation in terms of the convection problem. We shall first construct three intermediate problems that will allow us to analyse the roles of each of the new terms individually.

Subsystem A: What is the effect of the flow \( F(x) \) on the stability of RB system?

- Seek normal mode solutions of the form
  \[
  \varphi(x, y, t) = \Phi_0(x)e^{iky}e^{\imath \sigma t},
  \theta(x, y, t) = \Theta_0(x)e^{iky}e^{\imath \sigma t},
  \]
  where \( \sigma \) is the growth rate, and \( k \) is the wavenumber of the disturbance.
- For a particular \( F(x) \), solve the system (3), (4a) at each \( k \), to find the Ra number to the onset of instability (\( \sigma = 0 \)).
- Example results for flows \( F_1(x) = 1 \), \( F_2(x) = (2 - x)^{-1} \), \( F_3(x) = \cos(x) \), \( F_4(x) = -\sin(2\pi x) \) are shown in Figure 2.
- Clearly there is a preference of the directionality of \( F(x) \).

Q: Can we understand/predict what flows will be most stabilising/destabilising?

Perturbation analysis: consider RB system close to marginal stability and perturb it slightly, i.e. let \( F(x) = \epsilon f(x) \), with \( \epsilon \ll 1 \), and \( \sigma = \sigma_0 + \mathcal{O}(\epsilon^2) \).

Analytical expression for first order growth rate correction can be found

\[
\sigma_1 = -\frac{\epsilon}{\mathcal{F}} \frac{f}{\mathcal{F} - 1} \int \frac{f(x) \sin 2\pi x \, dx}{2\pi x}
\]

The analytical result correctly predicts the destabilising effect of \( F_2 \) and \( F_4 \), but has limitations, for example it fails to explain the stabilising effect of \( F_1 \) or \( F_3 \).

References


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